Maximum Economic Yield

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Abstract

Analytical results for steady-state values of the biomass that maximises the sum of inter-temporal economic profits (dynamic b_{MEY}) are derived in terms of generalised harvesting function. The conditions under which dynamic b_{MEY} exceeds the biomass that maximises the sustained yield (b_{MSY}) are evaluated under a range of conditions including when the discount rate exceeds the intrinsic growth rate, with a variable stock effect, technological change, and from an increase in the cost per unit of effort. The findings show that dynamic b_{MSY} provides both a sustainable and profitable management target under a wide range of parameter values.

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I. Introduction

The stock or biomass target that maximises the economic yield (b_{MEY}) has a long history in fisheries dating back to the seminal work of Warming (1911), Gordon (1954) and Scott (1955). Gordon (1954) showed that b_{MEY} would always be greater than the biomass that maximises the sustained yield b_{MSY}. Smith (1969) developed one of the first dynamic models in fisheries but assumed a zero discount rate and, consequently, also found that a private owner's b_{MEY} would always be greater than b_{MSY}. Clark (1973) and Clark and Munro (1975) and others developed steady-state expressions for dynamic b_{MEY} in an inter-temporal setting with discounting, but assumed that harvesting costs were proportional to the biomass. Clark (1973) showed that with a sufficiently high enough discount rate that it is possible for dynamic b_{MEY} to be less than b_{MSY}, and if the discount rate exceeded the intrinsic growth rate of the fishery, it is possible to have 'optimal' extinction with private ownership.

Despite the exhortations of fisheries economists over many decades, it is only recently that dynamic b_{MEY} has started to become accepted as an important and implementable target in fisheries management (Department of Agriculture, Fisheries and Forestry 2007; World Bank 2008). Recent developments in solving non-linear dynamic problems have also allowed the calculation of optimal transition paths to dynamic b_{MEY} for fishery managers. In four fisheries where dynamic b_{MEY} has been calculated, without assuming linearity in the cost function, it has been found that dynamic $b_{\mathrm{MEY}} > b_{\mathrm{MSY}}$ under reasonable discount rates (Grafton et al. 2007). These results show the prescience of Scott (1955, p. 123) who observed that "If increased output tends to diminish the population and so to reduce the net revenue earned in other periods had output been constrained today, … sole ownership would result in still greater reduction in desired output than would be the case if short run considerations only were at stake."

The empirical finding that $b_{\mathrm{MEY}} > b_{\mathrm{MSY}}$ is not a general result as the relative size of dynamic b_{MEY} to b_{MSY} will depend on the 'stock effect' or how sensitive are harvesting costs to reductions in the biomass or stock size, the intrinsic growth rate, the discount rate and output and input price parameters. The importance of the result is that there is a 'win-win' outcome from increasing the current level of the biomass (b_{CUR}) in terms of the size of the biomass and profitability whenever b_{CUR} < dynamic b_{MEY}.

Since the findings of Grafton et al. (2007) there have been a number of papers (Clark et al. in press, Christensen 2009) claiming the dynamic $b_{MEY} > b_{MSY}$ result is either incorrect and/or incomplete. It has also been claimed that dynamic b_{MEY} should either not be the biomass target for fisheries management or that it is equivalent to b_{MSY} (Christensen 2009). Given the importance of this debate for fisheries management and, in particular fisheries economics, this paper provides a detailed review of the conditions under which dynamic b_{MEY} may exceed b_{MSY} from the perspective of the harvesting sector.

For completeness, section II briefly reviews the static b_{MEY} result. The dynamic b_{MEY} result is then derived in section III and, for the first time, steady-state results are compared to b_{MSY} under a range of conditions derived using a non-linear cost function. Section IV briefly reviews the sensitivity of the results to a variable stock effect, technological change, and non-constant cost-price parameters. All results are derived analytically.

The contribution of the paper is to show: (1) that dynamic $b_{MEY} > b_{MSY}$ can exist under a wide range of conditions, including the case when the discount rate exceeds the intrinsic growth rate; (2) the need to appropriately estimate the stock effect when calculating dynamic b_{MEY} and not to assume harvesting costs are proportional to changes in the biomass. Overall, the paper supports the view that dynamic b_{MEY}, with appropriate sensitivity analysis, can generate a valuable and sustainable target for fisheries managers.

II. Static b_{MEY}

To provide a direct comparison to dynamic b_{MEY}, we formally derive the static b_{MEY} result. All results are obtained under the assumption of logistic growth, but dynamic b_{MEY}

can also be calculated with Ricker stock-recruitment relationships or more complicated age-cohort models (Quinn and Deriso 1999). Equation (1) specifies the growth function for the fishery,

$$g(b) = rb\left(1 - \frac{b}{K}\right) \tag{1}$$

where $g(b)$ is the growth in the biomass, b is the biomass, r is the intrinsic growth rate and K is the maximum carrying capacity of the single-species fishery. Growth in the biomass is maximised when $b = \frac{K}{2}$ and is denoted by b_{MSY}. The generalised harvest function for the fishing fleet is defined by equation (2) where h and b are specified in the same units,

$$h = q\left(\frac{b}{K}\right)^{\alpha} e^{\beta} \tag{2}$$

where h is harvest, q is a catchability coefficient that is analogous to total factor productivity for the fishery, e is a composite measure of fishing effort, α is a parameter that determines the 'stock effect' or how sensitive is the harvest to the size of the biomass, and β is a parameter that determines the marginal product of fishing effort. Without loss of generality K can be normalised to 1.0 such that $h = qb^{\alpha}e^{\beta}$. To ensure the existence of an equilibrium harvest we specify that $0 < \beta \leq 1.0$.

Solving (2) in terms of fishing effort and assuming fishers face a constant cost per unit of effort of $\breve{c}$ the harvesting cost function for the fishery is derived in equation (3),

$$C(b,h) = \breve{c}e(b,h,q) = \bar{c}q^{-\frac{1}{\beta}}b^{-\frac{\alpha}{\beta}}h^{\frac{1}{\beta}} \qquad (3).$$

Equation (3) can also be written in a more compact form if we define $\tilde{c} = \breve{c} q^{-\frac{1}{\beta}}$ such that $C(b,h) = \tilde{c} b^{-\frac{\alpha}{\beta}} h^{\frac{1}{\beta}}$.

Fishing profit is defined by equation (4) assuming the price of fish is a constant *P*,

$$\pi(b,h) = Ph - \tilde{c} b^{-\alpha/\beta} h^{1/\beta} \tag{4}$$

The profit-maximising level of the biomass is homogenous to degree zero with respect to the cost-price ratio $\frac{\tilde{c}}{P}$ and, thus, fishing profit can be redefined as,

$$\pi(b,h) = h - c b^{-\alpha/\beta} h^{1/\beta} \tag{5}$$

where $c = \frac{\tilde{c}}{P}$.

The analytical expression for the static $b_{\text{MEY},}$ given in equation (6), is obtained by maximising (5) subject to the constraint that the harvest equals the growth in the biomass.

$$\frac{c}{\beta}\left[rb(1-b)\right]^{\frac{1-\beta}{\beta}} = \frac{2b-1}{2b-1+\alpha(1-b)} b^{\frac{\alpha}{\beta}} \tag{6}$$

Over the domain $\left[\frac{1}{2},1\right)$, the left-hand side of equation (6) is a continuous, decreasing function (and strictly decreasing if $\beta < 1.0$) with respect to the biomass and is zero when $b = 1.0$ while the right-hand side of equation (6) is a continuous function increasing from 0 to 1 in the same domain. Provided that β and c are both strictly positive and less than

1.0, equation (6) ensures an interior solution where $b^* \in \left(\frac{1}{2}, 1.0\right)$ such that static b_{MEY} > b_{MSY}. If $c = 0$ then static b_{MEY} = b_{MSY}.

III. Dynamic b_{MEY}

The dynamic optimisation problem in continuous time with a strictly positive discount rate ρ is defined as follows:

$$V(b_0) = Max_{\{h(t)\}} \int_0^\infty \pi(b,h) e^{-\rho t} dt \text{ subject to: } \begin{cases} \dot{b} = rb(1-b) - h \\ b(0) = b_0 \\ 0 \le b(t) \le 1 \end{cases}$$

where b_0 is the initial value of the biomass and both b and h are functions of time. Provided that $0 < \beta \le 1.0$ the profit function is concave with respect to both b and h and the second-order conditions for profit maximisation are satisfied. An expression for the steady-state biomass can be derived using the maximum principle as follows:

$$\frac{c}{\beta}\left[rb(1-b)\right]^{\frac{1-\beta}{\beta}} = \frac{\rho - r + 2rb}{\alpha r(1-b) + (\rho - r + 2rb)} b^{\frac{\alpha}{\beta}} \qquad (7)$$

Provided that equation (7) has a real root in the open interval (0, 1) there exists a strictly positive value for the steady-state level of the biomass b^*. In general, an increase in ρ will reduce b_{MEY}. This is because the left-hand side of equations (6) and (7) are identical such that dynamic b_{MEY} > static b_{MEY} if $\rho > 0$ when there exists an interior solution.

The relative magnitude of dynamic b_{MEY} to b_{MSY} will depend on the catchability coefficient and the cost-price parameters (c), the harvest function parameters (α and β), the discount rate (ρ) and the intrinsic growth rate (r). We present possible values for dynamic b_{MEY} under two scenarios: one, the discount rate is strictly less than the intrinsic growth rate ($\rho < r$) and, two, the discount rate is strictly greater than the intrinsic growth rate ($\rho > r$).

Case $\rho < r$

If the cost-price ratio (c) is zero then from equation (7) dynamic $b_{\text{MEY}} = \frac{1}{2} - \frac{\rho}{2r}$ where the term $\frac{\rho}{2r}$ represents the impatience that arises from a positive discount rate. This implies that dynamic $b_{\text{MEY}} < b_{\text{MSY}}$.

If $c > 0$ and $\beta \in (0,1.0)$ then there exists a value for b_{MEY} in the interval defined by $b_{\text{MEY}} \in \left(\frac{1}{2} - \frac{\rho}{2r}, 1\right]$. This is because the left-hand side of equation (7) is increasing in the interval in $\left(\frac{1}{2} - \frac{\rho}{2r}, \frac{1}{2}\right]$ and decreasing to zero in $\left(\frac{1}{2}, 1\right]$ while the right-hand side of equation (7) is a continuous function increasing from 0 to 1 in the domain $\left(\frac{1}{2} - \frac{\rho}{2r}, 1\right]$. Thus, there is a steady-state solution in terms of b in the specified domain, that is, $\frac{1}{2} - \frac{\rho}{2r} \leq$ dynamic $b_{\text{MEY}} <$ static $b_{\text{MEY}} \leq 1.0$. In other words, the dynamic b_{MEY} is strictly positive, but may be greater, equal to or less than b_{MSY} depending on the value of the parameters given in equation (7).

Case $\rho > r$

The optimal extinction result, first shown by Clark (1973), exists when the cost-price ratio (c) is zero and may also arise when $c > 0$ if the stock effect is sufficiently small such that $\alpha+\beta \le 1.0$ and c is equal to or less than the cost-price threshold ($\hat{c}$) given by equation (8). If $\alpha+\beta=1.0$, the cost threshold is defined by,

$$\hat{c}=r\left(\beta\frac{\rho-r}{\alpha r+\rho-r}\right)^{\frac{\beta}{1-\beta}} \qquad (8).$$

Equally as important is the case when dynamic b_{MEY} is strictly positive. As far as we are aware this result, until now, has not been derived analytically. If $c > 0$, $\beta<1.0$ and the stock effect is sufficiently large such that $\alpha+\beta>1.0$ then dynamic $b_{\text{MEY}} > 0$. This result can be shown by rearranging equation (7) as follows:

$$c\left(1-b\right)=r\left(\beta\frac{\rho-r+2rb}{\alpha r\left(1-b\right)+\left(\rho-r+2rb\right)}\right)^{\frac{\beta}{1-\beta}} b^{\frac{\alpha+\beta-1}{1-\beta}} \qquad (9)$$

The right-hand side of equation (9) is a continuous function that is strictly increasing from 0 to $r\beta^{\frac{\beta}{1-\beta}}$ over the domain (0, 1) while if $c > 0$ then the left-hand side of equation (9) is linear and downward sloping. Thus, there exists an interior solution in terms of b.

A strictly positive dynamic b_{MEY} also exists when $\beta=1.0$ provided that $c < 1.0$ while the fishery is unexploited if $c \ge 1.0$. This can be shown by rearranging equation (7) as follows:

$$c=\frac{\rho-r+2rb}{\alpha r\left(1-b\right)+\left(\rho-r+2rb\right)}b^{\alpha} \qquad (10)$$

The right-hand side of equation (10) is a continuous function that is strictly increasing from 0 to 1.0 over the domain (0, 1.0). Thus, provided $c < 1.0$ there is an interior solution in terms of b.

Not only can we establish that dynamic b_{MEY} can be strictly positive even when the discount rate is greater than the intrinsic growth rate, we can also derive a sufficient condition that ensures that dynamic $b_{\text{MEY}} > b_{\text{MSY}}$. The sufficient condition is given by expression (11) below, provided that $c > 0$ and $\beta \le 1.0$,

$$c > \beta \left(r/4 \right)^{1-\frac{1}{\beta}} \qquad (11).$$

If $\beta = 1.0$ and $c > 1.0$ the fishery is unexploited. If $\beta < 1.0$, the sufficient condition given by eexpression (11) can be derived from equation (7) where the left-hand side is continuous and strictly decreasing over the domain $\left(\frac{1}{2}, 1 \right)$. Given expression (11), the left-hand side of equation (7) is greater than 1.0 if $b = \frac{1}{2}$ and is zero when $b = 1$. The right-hand side of equation (7) is also a continuous function over the same domain, but strictly increasing from a point below 1.0.   Thus, there must be an interior solution in the domain $b \in \left(\frac{1}{2}, 1 \right)$ such that dynamic $b_{\text{MEY}} > \frac{1}{2}$. This is a striking result because it is independent of the discount rate. This does *not* imply that the discount rate does not affect dynamic b_{MEY} , but given (11), it does imply that the catchability coefficient, the cost-price parameters and the harvest function parameters are such that dynamic $b_{\text{MEY}} > b_{\text{MSY}}$.

IV. Dynamic b_{MEY} under Alternative Scenarios

In this section the effects on dynamic b_{MEY} from changes in the stock effect, technological change, and the cost-price parameters are explored.

Variable stock effect

The stock effect, as represented by the parameter α need not necessarily be a constant, or be independent of the biomass. It is possible that as the biomass declines that the stock effect increases such that $\alpha(b) > 0$ and $\alpha'(b) < 0$. Under this assumption the previous results that ensure a strictly positive dynamic b_{MEY} still hold, but under the following modified condition:

$$\lim_{b \to 0} \alpha(b) + \beta > 1 \qquad (12)$$

Cost-price parameters

Using equation (9), and provided there exists a strictly positive dynamic $b_{MEY,}$ it can be shown that an increase in *c* will increase dynamic b_{MEY}. This is because to ensure equality of equation (9), for an autonomous increase in *c*, *b* must also be larger.

Technological change

Improvements in technology that reduce the cost of harvesting fish for a given biomass may be represented by an increase in the catchability coefficient (*q*). An increase in *q* reduces *c*, all else equal, and thus lowers dynamic b_{MEY}. Technological change may also lead to optimal extinction under the following conditions: (i) the discount rate is larger than the growth rate; (ii) the stock effect is always weak such that $\alpha(b) + \beta < 1$ for all *b*, and (iii) $c < \hat{c}$.

V. Concluding Remarks

The biomass that maximises the economic profit from a fishery (dynamic b_{MEY}) has long been recommended by economists as a management target. It has the potential to generate a 'win-win' that increases both economic profits and the size of the fishery whenever the current biomass is less than dynamic b_{MEY}.

Dynamic b_{MEY} is increasingly being used as a management target, but it has also become the object of criticism by both biologists and economists. The critiques are that: (i) the dynamic b_{MEY} target is incomplete or insufficient to account for values beyond the harvesting sector; and (ii) in fisheries where the intrinsic growth rate is less than the discount rate it will result in optimal extinction. While by no means a complete analysis, this paper responds to these criticisms by providing, for the first time, analytical results that show the adoption of dynamic b_{MEY} as a management goal need not result in extinction even when the discount rate exceeds the intrinsic growth rate. The results also show under what conditions dynamic b_{MEY} exceeds the biomass that maximises the sustained yield even when the discount rate exceeds the intrinsic growth rate.

Overall, the analysis shows the importance of empirically estimating generalised harvesting functions in fisheries so as to calculate the parameters required to estimate dynamic b_{MEY}. The findings indicate that, with appropriate sensitivity analysis, dynamic b_{MEY} is a valuable management target that is sustainable under a wide range of parameter values.

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